

## Set notation

- ▶ A set is a collection of objects called elements.

If  $a$  is an element of the set  $S$ , we write

$$a \in S,$$

and say that  $a$  belongs to  $S$ . If  $b$  does not belong to  $S$ , we write  $b \notin S$ .

- ▶ Sets can be specified in two ways:

(a) Listing its elements

$$A = \{x_1, x_2, \dots, x_n\} \quad \text{finite}$$

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$$B = \{x: \underbrace{1 < x < 2}_{\text{infinite sets}}\}$$

- ▶ The symbol  $\emptyset$  is used to denote the empty set.

$$\{x: x^2 + 1 = 0\} = \emptyset$$

$$x^2 + 1 = 0 \Rightarrow x^2 = -1$$

- ▶ Let  $S$  and  $T$  be two sets. If every element of  $S$  also belongs to  $T$ , we say that  $S$  is a subset of  $T$  and write

$$S \subseteq T.$$

Every  $a \in S$  also  $a \in T$



- ▶ Two sets  $A$  and  $B$  are equal, denoted by

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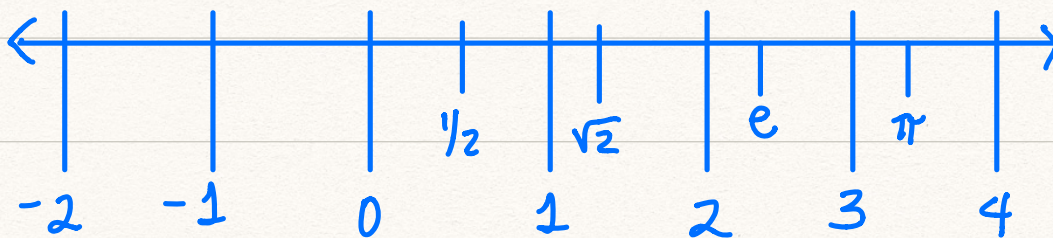
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## The set of real numbers $\mathbb{R}$ .

Usually represented as a straight, solid line that extends indefinitely in both directions.



## Arithmetic

For every  $a, b, c \in \mathbb{R}$ , the following properties hold:

	Addition:	Multiplication:
<b>Commutativity:</b>	$a + b = b + a$	$ab = ba$
<b>Associativity:</b>	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c = abc$
<b>Identity:</b>	There is a real number 0, such that $a + 0 = a$ .	There is a real number 1, such that $1a = a$ .

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Subtraction and division are defined as

$$a - b = a + \underset{\substack{\uparrow \\ \text{additive} \\ \text{inverse of } b}}{(-b)} \quad \text{and} \quad \frac{a}{b} = a \underset{\substack{\uparrow \\ \text{multiplicative inverse}}}{b^{-1}} \quad (b \neq 0)$$

where  $-b$  and  $b^{-1}$  are the additive and multiplicative inverse of  $b$ , respectively.

▶ Division by 0 is not defined. The expression  $\frac{2}{0}$  makes no sense.

▶  $\infty$  is not a real number and it is not true that  $\frac{2}{0} = \infty$ .

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## Theorem

The additive and multiplicative identities are unique.

Proof: Suppose there is some  $u \in \mathbb{R}$  such that  $au = a$  for all  $a \in \mathbb{R}$

We will prove that  $u = 0$ .

$$au = a \quad \forall a \in \mathbb{R}$$

$$a = 0 \quad u = 0 + u = 0$$

Suppose that there is some  $v \in \mathbb{R}$  such that  $va = a^*$   $\forall a \in \mathbb{R}$

Let  $\mathbb{I}$  fix  $a = 1$

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## Theorem

Let  $a$  be any real number. Then  $a$  has a unique additive inverse. If  $a \neq 0$ , it has a unique multiplicative inverse.

Proof: Fix  $a \in \mathbb{R}$ .  $(-a)$   $a + (-a) = 0$

Suppose that  $u$  is also an additive inverse of  $a$ . We will prove that  $u = -a$

$$u = u + 0 = u + [a + (-a)] \stackrel{\text{Associativity}}{=} (u + a) + (-a)$$
$$(u + a = 0) = 0 + (-a) = -a \Rightarrow \boxed{u = -a}$$

Suppose that  $v$  is also a mult. inverse of  $a$ . ( $a^{-1}$  and  $a \cdot a^{-1} = 1$ )

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## Theorem

For any  $x \in \mathbb{R}$ , if  $a+x=b+x$ , then  $a=b$ .  
(hyp)

Proof:

$$a = a + 0$$

$$= a + (x - x)$$

$$= (a+x) - x$$

$$\text{(hyp.)} = (b+x) - x$$

$$= b + (x - x)$$

$$= b + 0$$

$$= b$$

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## Theorem

1. For any non-zero  $x \in \mathbb{R}$ , if  $ax = bx$ , then  $a = b$ .

2.  $0x = 0$  for all  $x \in \mathbb{R}$

3.  $1 \neq 0$ .

4.  $(-1)x = -x$  for all  $x \in \mathbb{R}$ .

5.  $-(-x) = x$  for all  $x \in \mathbb{R}$ .

6. If  $xy = 0$ , then either  $x = 0$  or  $y = 0$ .

7. For all  $x, y \in \mathbb{R}$ ,  $x(-y) = -(xy)$ .

8. For all  $x, y \in \mathbb{R}$ ,  $(-x)(-y) = xy$ .

9. If  $x \neq 0$ , then  $x^{-1} \neq 0$  and  $(x^{-1})^{-1} = x$ .

10. If  $x \neq 0$  and  $y \neq 0$ , then  $xy \neq 0$  and  $(xy)^{-1} = x^{-1}y^{-1}$ .

11. For any non-zero  $x \in \mathbb{R}$ ,  $(-x)^{-1} = -x^{-1}$

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